A Conjecture on Minimum Multiplicity for Kirchhoff Graphs

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Kirchhoff graphs are a type of vector graph where the vertex cuts (incidence vectors) and cycle vectors satisfy an orthogonality condition: $RN = 0$ where the rows of R are a cut-space basis and the columns of N are a cycle basis. Given such a matrix, R, we can construct Kirchhoff graphs that satisfy this condition using the columns of R as edge vectors. All Kirchhoff graphs that are vector 2-connected must contain the same number of copies of each edge vector; define m to be this multiplicity. Since the same matrix may give rise to many Kirchhoff graphs, we are interested in finding the minimum multiplicity m^* for which there is a nontrivial Kirchhoff graph. We conjecture that if R is augmented with additional rows containing certain patterns of zeros, then the LCM of the absolute values of the elements of the resulting matrix will be m^*.