Intersection of Longest Cycles and Largest Bonds in 3-Connected Graphs

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A bond in a graph is a minimal nonempty edge-cut. A connected graph G is dual Hamiltonian if the vertex set can be partitioned into two subsets X and Y such that the subgraphs induced by X and Y are both trees. There is much interest in studying the longest cycles and largest bonds in graphs. A. Sanford determined in her Ph.D. thesis the cycle spectrum of the well-known generalized Petersen graph $P(n, 2)$(n is odd) and $P(n, 3)$(n is even). M. Flynn proved in her honors thesis that any generalized Petersen graph $P(n, k)$ is dual Hamiltonian. H. Wu conjectured that any longest cycle must meet any largest bond in a 3-connected graph.
In this research, I first study the bond spectrum (called the co-spectrum) of the generalized Petersen graphs and extend Flynn's result by proving that in any generalized Petersen graph $P(n, k), 1 \leq k < n/2$, the co-spectrum of $P(n, k)$ is $\{3, 4, 5, \ldots, n + 2\}$. In my second result, the core part of this research, I prove that the above conjecture is true for certain classes of 3-connected graphs: Let G be a 3-connected graph with n vertices and m edges. Suppose $c(G)$ is the size of a longest cycle, and $c^*(G)$ is the size of a largest bond. Then each longest cycle meets each largest bond if either $c(G) \geq n - 3$ or $c^*(G) \geq m - n - 1$.

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