The degree/Steiner k-diameter problem for trees

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Given integers $d \geq 1$ and $\Delta \geq 3$, *the degree/diameter problem* asks for the largest possible order of a graph with diameter d and maximum degree Δ. It was shown in [Christou, Iliopoulos, and Miller, Degree/diameter problem for trees and pseudotrees. *AKCE International Journal of Graphs and Combinatorics*, 10:4, 377-389 (2013)] that for any tree of order n, diameter d, and maximum degree Δ, we have that

$$n \leq \begin{cases} \dfrac{2(\Delta-1)^{(d+1)/2}-2}{\Delta-2}, & \text{if } d \text{ is odd, and} \\ \dfrac{\Delta(\Delta-1)^{d/2}-2}{\Delta-2}, & \text{if } d \text{ is even.} \end{cases}$$

We extend the degree diameter problem to *the degree/Steiner k-diameter problem.* That is, given integers $k \geq 2$, $d \geq k-1$, and $\Delta \geq 3$, we ask for the largest possible order of a graph with maximum degree Δ and Steiner k-diameter d. Furthermore, we show that if a tree has order n, diameter d, and maximum degree Δ, then $n \leq \dfrac{\Delta(\Delta-1)^{\ell}-2}{\Delta-2} + (d-k\ell)(\Delta-1)^{\ell}$, where $\ell = \min\left\{ \left\lfloor \dfrac{d}{k} - \dfrac{k-\Delta}{k(\Delta-2)} \right\rfloor, \left\lfloor \dfrac{d}{k} \right\rfloor \right\}$. Furthermore, we show that this bound is sharp.

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