On Forward Difference of Surface Areas of Interconnect Networks

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For any node u in a graph G and integer i, the surface area centered at u with distance i is defined to be the number of nodes in G whose distance to u is i. If the graph G is the topology of a symmetric interconnection network in parallel computing, the applications of the surface area of G include computing bounds for some communication algorithms. One approach to finding a closed-form formula to the surface area of an interconnection network is to use forward differences provided that the forward differences of the surface area satisfy certain conditions, namely, for the surface areas of distance i, their $(i+1)$-th forward difference is 0. We will first review how forward differences are used to obtain closed-form solutions of surface areas. We then show that for certain networks, if a recurrence is given for its surface area, we can prove that its surface area satisfies this forward difference property. These networks include the hypercube, augmented cube, and the (n,k)-star. We believe that this approach of using a recurrence of the surface area of the network to show the forward difference property applies to many other networks.

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