

On CNF Base Hypergraphs

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A CNF base hypergraph (BHG) has the propositional variables of a CNF formula (or clause set) as its vertex set; the edge set consists of all bases, i.e., the variable sets underlying the CNF clauses. All clauses of the formula having the same base form a fibre. The BHG can be viewed as the projecting base of the CNF; and so it represents the base structure of a large amount of formulas simultaneously, namely all those having the same BHG. A BHG is diagonal if it admits an unsatisfiable (fibre-)transversal, which is a formula that contains exactly one clause of each fibre for all bases of the BHG. We discuss the basic properties of (diagonal) BHGs and provide several examples. The class of diagonal BHGs decomposes into subclasses according to the number of orbits that arise in the space of all diagonal transversals via the action of the group of variable complementation. We address the question whether for every positive integer such a subclass exists. We also discuss the (dense) maximal non-diagonal BHGs whose members are only one hyperedge away from diagonality. Several concrete classes of that property are constructed. We also provide non-diagonal BHGs based on finite projective planes that become minimal diagonal by a retraction. Via a non-commutative joining operation one is enabled to construct arbitrary large uniform minimal diagonal BHGs.

Keywords: hypergraph, CNF-satisfiability, orbit, transversal, finite-projective-plane