Cyclic Sieving Phenomenon for Fans of Dyck Paths

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The cyclic sieving phenomenon is a triple $(X, C_n, f(q))$ where X is a set having a cyclic group C_n -action and f(q) is a function that when evaluated at the *n*-th roots of unity keeps track of the number of fixed elements of X under this action. We construct an injection from the set of *r*-fans of Dyck paths of length *n* into the set of chord diagrams on [n] that intertwines promotion and rotation. Using this correspondence, we show that *r*-fans of Dyck Paths exhibit the cyclic sieving phenomenon. Moreover, this injection provides a rotational invariant diagrammatic basis for a certain type B_r representation.

Keywords: Dyck paths, chord diagrams, cyclic sieving, promotion, crystal graphs