

## Harmonious Labelings of Symmetric Starlike Trees

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We define a symmetric starlike tree as a tree where each branch of the tree is built by overlaying the center of an odd path,  $P_r$ , on all but one of the end vertices of a path. We then inductively continue to overlay the center of odd paths of the same length to each additional vertex obtained from the previous step. The tree is called  $n$ -dimensional, if this process is done  $n - 1$  times. Finally, we connect each vertex of degree one for all branches together, which is called the root of the tree. It has recently been shown that one and two dimensional symmetric starlike trees with an odd number of branches are harmonious (Landers, Mattox, Painter). Two-dimensional symmetric starlike trees are defined as snowflake graphs. This talk will generalize harmonious labelings of snowflake graphs to  $n$ -dimensional symmetric starlike trees with an odd number of branches. We achieve these generalized harmonious labelings using a new group operation on the direct product  $\mathbb{Z}_{m_1} \times \cdots \times \mathbb{Z}_{m_q}$  that forces this direct product to always be isomorphic to  $\mathbb{Z}_m$ , where  $m = m_1 \cdots m_q$ , regardless of the factorization of  $m$ .

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