Maximizing Subgraphs in Regular Graphs

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We determine the d-regular graphs G on n vertices with the highest H-density for any H. We reframe the problem as a continuous optimization problem on the eigenvalues of G by relating injective homomorphism numbers from H and homomorphism numbers from quotient graphs of H. For almost all H, this relation has *non-spectral terms*, which require bounding by *spectral terms* in a way that is sharp at the optimal graph. As such, we establish an infinite family of homomorphism number inequalities.

For bipartite H and d large enough, we show G consists of disjoint copies of $K_{d,d}$. For non-bipartite H and d sufficiently large, G is a collection of disjoint K_{d+1} graphs. For $H = C_5$ and $d = 3$, disjoint Petersen graphs emerge.

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