Regular graphs with two distinct eigenvalues

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For an $n \times n$ matrix A, let $q(A)$ be the number of distinct eigenvalues of A. If G is a connected graph on n vertices, let $\mathcal{S}(G)$ be the set of all real symmetric $n \times n$ matrices $A = [a_{ij}]$ such that $a_{ij} = 0$ if and only if $\{i, j\}$ is not an edge of G. Let $q(G) = \min\{q(A)|A \in \mathcal{S}(G)\}$. Determining $q(G)$ is a fundamental subproblem of the inverse eigenvalue problem for graphs, and characterizing the case for which $q(G) = 2$ has been especially difficult. Some progress has been made by considering the problem for regular graphs. It turns out that it is easy if the degree of regularity is $1, 2$, or 3. But the $4-$regular graphs with $q(G) = 2$ are much more difficult to characterize. A connected $4-$regular graph has $q(G) = 2$ if and only if either G belongs to an easily described infinite class of graphs, or else G is one of sixteen $4-$regular graphs whose number of vertices ranges from 5 to 16. This technical result gives rise to several intriguing question.