C_4-Face-Magic on a 4×4 Klein Bottle Grid Graph

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For a graph $G = (V, E)$ embedded in the Klein bottle, let $\mathcal{F}(G)$ denote the set of faces of G. We will call G a C_4-face-magic Klein bottle graph if there is a bijection $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ such that for any $F \in \mathcal{F}(G)$ with $F \cong C_4$, the sum of all the vertex labelings along C_4 is a constant S called the face magic value of G. Let $x_v = f(v)$ for all $v \in V(G)$. We shall call $\{x_v : v \in V(G)\}$ a C_4-face-magic Klein bottle labeling on G.

In this talk we will consider the 4×4 grid graph, denoted $\mathcal{K}_{4,4}$, that is embedded in the Klein bottle in the natural way, which will have a face magic value of $S = 34$. We will say that a C_4-face-magic Klein bottle labeling $X = \{x_{i,j} : (i, j) \in V(\mathcal{K}_{4,4})\}$ on $\mathcal{K}_{4,4}$ is vertically pairwise balanced if $x_{i,2j-1} + x_{i,2j} = \frac{1}{2}S$ for all $1 \leq i \leq 4$ and $1 \leq j \leq 2$; and is horizontally pairwise balanced if $x_{2i-1,j} + x_{2i,j} = \frac{1}{2}S$ for all $1 \leq i \leq 2$ and $1 \leq j \leq 4$.

We will define and apply elementary vertical cycle operations to a standard C_4-face-magic labeling of $\mathcal{K}_{4,4}$ that is either horizontally or vertically pairwise balanced in order to count the total number of C_4-face-magic labelings on $\mathcal{K}_{4,4}$ (up to symmetries on the Klein bottle).

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