Uncommon Systems of Equations

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A system of linear equations L over $\mathbb{F}_q$ is *common* if the number of monochromatic solutions to L in any two-colouring of $\mathbb{F}_q^n$ is asymptotically at least the expected number of monochromatic solutions in a random two-colouring of $\mathbb{F}_q^n$. Motivated by existing results for specific systems (such as Schur triples and arithmetic progressions), as well as extensive research on common and Sidorenko graphs, the systematic study of common systems of linear equations was recently initiated by Saad and Wolf. Building on earlier work of of Cameron, Cilleruelo and Serra, as well as Saad and Wolf, common linear equations have been fully characterised by Fox, Pham and Zhao.

In this talk I will discuss some recent progress towards a characterisation of common systems of two or more equations. In particular we prove that any system containing an arithmetic progression of length four is uncommon, confirming a conjecture of Saad and Wolf. This follows from a more general result which allows us to deduce the uncommonness of a general system from certain properties of one- or two-equation subsystems.