Longest Path and Cycle Transversals in Chordal Graphs

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A *longest path (cycle) transversal* in a graph G is a set of vertices that intersects each longest path (cycle) in G, and the *longest path (cycle) transversal* number of G, denoted $\mathrm{lpt}(G)$ ($\mathrm{lct}(G)$) is the minimum size of a longest path (cycle) transversal. In 1968, Gallai asked if every connected graph G satisfies $\mathrm{lpt}(G) = 1$. This is false in general but is true when G is restricted to many natural graph subclasses, such as interval graphs (Balister, Győri, Lehel, Schelp [2004]), circular arc graphs (Joos [2015]), and several graph classes defined by forbidding a particular induced subgraph. Balister, Győri, Lehel, and Schelp [2004] asked if $\mathrm{lpt}(G) = 1$ when G is a connected chordal graph. This question remains open. Harvey and Payne [2023] proved that $\chi(G) \leq 4\lceil \omega(G)/4 \rceil$ when G is a connected chordal graph, where $\omega(G)$ is the maximum size of a clique in G. We obtain upper bounds on $\mathrm{lpt}(G)$ and $\mathrm{lct}(G)$ in terms of n when G is a connected n-vertex chordal graph.

Keywords: longest path, transversal, chordal graph