**Dual min-max graph combinatorial problems for multi-dimensional grid networks**

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Dual min-max graph combinatorial problems are a pair of combinatorial problems where the maximum value in the first combinatorial problem is less than or equal to the minimum value in the second problem. The advantage of dual min-max graph combinatorial problems is that the concepts of the first combinatorial problem will be used to solve the second problem. The minimum vertex cover problem and the maximum matching problem are well-known dual min-max graph combinatorial problems. The edge geodesic cover problem of a graph $G(V, E)$ is to find a smallest number of geodesics that cover the edge set $E(G)$ of $G$. The edge k-general position problem is to find a largest set $S$ of edges of $G$ such that no $k-1$ edges of $S$ lie on a common geodesic. Recently, a new pair of dual min-max invariant combinatorial problems which are the edge geodesic cover problem and the edge k-general position problem have been studied. After solving this new pair of dual min-max invariant combinatorial problems for different networks including torus networks, hypercubes, and Benes networks, it is stated that this new pair of dual min-max invariant combinatorial problems remains open for $r$-dimensional grids for $r \geq 3$. In this paper, this open problem is completely solved.

Keywords: geodesic cover problem, general position problem, multi-dimensional grid networks, algorithms and complexity;