Stochastic Strategies for Rendezvous Search on Platonic Solids

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Rendezvous search problems concern the optimal strategies for agents randomly placed in a space to find each other in the minimum expected time $\mathbb{E}(T)$. We consider a discrete model on the vertices of platonic solids and examine two cases: (1) when agents can see all vertices with whom they share a face, and (2) when agents can see all adjacent vertices. We contrast deterministic strategies, in which agents follow a predetermined sequence of oriented steps, with stochastic strategies that admit randomness at each move.

The simplest stochastic strategy is a random walk on the solid, which can be generalized to admit a probability of waiting at each step. It is known that for the cube in (1), the optimal deterministic and optimal random walk strategies share expected time $\mathbb{E}(T) \approx 1.333$. We introduce the first example of a stochastic strategy that combines random and deterministic steps in sequence, then suggest it optimal among all stochastic strategies on the cube in (1), with $\mathbb{E}(T) \approx 1.264$. We show analogous results on the octahedron in (1) and (2).

In contrast, we prove no deterministic strategy on the cube in (2) guarantees meeting in finite time. The best known strategy is a random walk with a predetermined waiting probability of $\alpha = 0.25$ and $\mathbb{E}(T) \approx 2.667$. We prove this strategy optimal among all random walk strategies in which α is selected from a $[0, 1)$-valued random variable. Finally, we consider future directions in using Bayesian approaches to inform stochastic strategies.

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