On 1-König-Egerváry graphs

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Let $\alpha(G)$ denote the cardinality of a maximum independent set, while $\mu(G)$ be the size of a maximum matching in the graph $G = (V, E)$. If $\alpha(G) + \mu(G) = |V|$, then G is a *König-Egerváry graph*, while if $\alpha(G) + \mu(G) = |V| - 1$, then G is a 1-*König-Egerváry graph*. If G is not König-Egerváry, but there exists a vertex $v \in V$ (an edge $e \in E$) such that $G - v$ ($G - e$) is König-Egerváry, then G is called a vertex almost König-Egerváry graph (an edge almost König-Egerváry graph, respectively). We characterize 1-König-Egerváry graphs, vertex (edge) almost König-Egerváry graphs, and present interrelationships between them. The *critical difference* $d(G)$ is $\max\{d(X) = |X| - |N(X)| : X \subseteq V\}$. The set A is *crtical*, if $d(A) = d(G)$. Let $\varrho_v(G)$ denote the number of vertices $v \in V$, such that $G - v$ is a König-Egerváry graph. We show that if G is a 1-König-Egerváry graph, then $\varrho_v(G) \leq n(G) + d(G) - \xi(G) - \beta(G)$, where $\xi(G)$ is the size of the intersection of all maximum independent sets, $\beta(G)$ is the size of union of all maximum crtical independent sets. As an application, we characterize the 1-König-Egerváry graphs that become König-Egerváry after deleting any vertex.

Keywords: maximum independent set, critical independent set, maximum matching, König-Egerváry graph.