Euler's Formula for General Graph Imbeddings

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Consider an imbedding of a graph $G(v, e)$ with $v \geq 1$ vertices and $e \geq 0$ edges into a closed surface s, with r resulting regions. If G is connected and every region is a 2-cell (a so-called 2-cell imbedding), Euler's formula is the relation $v - e + r = \chi(s)$, where $\chi(s)$ denotes the Euler characteristic of s. Here we give a generalization of Euler's formula which applies to any imbedding (2-cell or not) of any graph (connected or not) into any surface (orientable or not), with several interesting corollaries. One rather striking corollary is the converse of Euler's formula itself: If an imbedding of a graph $G(v, e)$ into a closed surface s merely has $r = e - v + \chi(s)$ regions, the right number for a 2-cell imbedding, it *is* a 2-cell imbedding.

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