Construction of fixed even size graphs with local antimagic chromatic number 3 - matrix & vertices merging approaches

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An edge labeling of a connected graph $G = (V, E)$ is said to be local antimagic if it is a bijection $f : E \to \{1, \dots, |E|\}$ such that for any pair of adjacent vertices x and y, $f^+(x) \neq f^+(y)$, where the induced vertex label $f^+(x) = \sum f(e)$, with e ranging over all the edges incident to x. The local antimagic chromatic number of G, denoted by $\chi_{la}(G)$, is the minimum number of distinct induced vertex labels over all local antimagic labelings of G. Suppose $\chi_{la}(G) = \chi_{la}(H)$ and G_H is obtained from G and H by merging some vertices of G with some vertices of H bijectively. In this paper, we first give ways to construct matrices with integers in $[1, 10k]$, $k \geq 1$, that meet certain properties. These matrices are then used to construct various families of graphs of size $10k$ with a corresponding local antimagic labeling. We then introduce the vertices merging approach to construct new families of graphs of size $10k$ with a corresponding local antimagic labeling. Consequently, we proved that all these (possibly disconnected) tripartite (and bipartite) graphs have local antimagic chromatic number 3. Open problems are also introduced.

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