A Nordhaus-Gaddum type problem for the normalized Laplacian spectrum and graph Cheeger constant

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Nordhaus-Gaddum type results relate invariants of a graph G with those of its complement G^c. The Cheeger constant $h(G)$ of a graph measures, in a sense, how evenly one could split a graph in two pieces while minimizing the number of edges removed. The normalized Laplacian matrix gives information about graph structure and random walks on the graph. Let $\lambda_2(G)$ denote the second smallest eigenvalue of the normalized Laplacian matrix. We show that $\max\{h(G), h(G^c)\} \geq \frac{2}{n}$ which implies $\max\{\lambda_2(G), \lambda_2(G^c)\} \geq \frac{2}{n^2}$. We give partial characterizations for graphs which achieve $\max\{h(G), h(G^c)\} = \frac{2}{n}$ and conjecture a stronger lower bound on $\max\{\lambda_2(G), \lambda_2(G^c)\}$.

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