Generating Cyclically 4-Connected Cubic Graphs

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A 3-connected cubic graph is cyclically 4-connected if it has at least $n \ge 8$ vertices and when removal of a set of three edges results in a disconnected graph, only one component has cycles. By introducing the notion of cycle spread to quantify the distance between pairs of edges, we get a new characterization of cyclically 4-connected graphs. Let Q_n and V_n denote the ladder and Möbius ladder on $n \ge 8$ vertices, respectively. We proved that a 3-connected cubic graph G is cyclically 4-connected if and only if G is either the Petersen graph, Q_n or V_n for $n \ge 8$, or G is obtained from Q_8 or Q_{10} by bridging pairs of edges with cycle spread at least (1, 2). The focus in this talk is the algorithmic nature of the results. The concept of cycle spread naturally leads to methods for constructing cyclically k-connected cubic graphs from smaller ones. We implemented an algorithm based on these results using McKay's nauty system for isomorphism checking.

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