Approximating Bimatrix Nash Equilibrium via Trilinear Minimax

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Computing a Nash equilibrium of a bimatrix game with $m \times n$ payoff matrices R and C is PPAD-complete and is unknown to be computable in polynomial time. While von Neumann's minimax theorem is a corollary of Nash equilibrium, we introduce a *trilinear minimax relaxation* (TMR) that yields efficiently computable approximations to bimatrix Nash equilibria. For mixed strategies $p = (x, y)$ in the product of unit simplices, let $R[p] = x^T Ry$ and $C[p] = x^T Cy$ denote the players' expected payoffs. We show that the minimax value λ^* of TMR is computable as a linear program in $O(mn)$ time and provides an upper bound on $\min\{R[p_*], C[p_*]\}$ over all Nash equilibria p_*. The optimal solution of the linear program yields a probability matrix whose row and column sums define an approximate Nash equilibrium p^*. We show that $\lambda^* \geq \lambda_*$, the maximin value of TMR, and that equality holds if and only if $\lambda^* = \min\{R[p^*], C[p^*]\}$. In this case, at least one player's payoff improves, and since Nash equilibria are generally Pareto-inefficient, both players' payoffs may improve. When $\lambda^* \neq \min\{R[p^*], C[p^*]\}$, we construct from p^* an alternative approximation $\widehat{p}^*$ such that for some $r^* \in [1, 2)$, $r^* \min\{R[\widehat{p}^*], C[\widehat{p}^*]\}$ upper-bounds $\min\{R[p_*], C[p_*]\}$ for any Nash equilibrium p_*. These results support TMR as a worthy relaxation, independent of the computation of an exact Nash equilibrium. We also present computational illustrations and extend the framework to the case of three or more players, giving rise to challenging open problems.

Keywords: Nash equilibrium, von Neumann minimax, linear programming