Babai Numbers and Spectra of Paths and Cycles

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Suppose that (X, ρ) is a metric space and $D \subset (0, \infty)$. The distance graph G(X, D) has vertex set X with x, y adjacent $\Leftrightarrow \rho(x, y) \in D$. The chromatic number of G(X, D) will be denoted $\chi(X, D)$.

Let $R(X, \rho) = \rho(X \times X) \setminus \{0\}$ the range of the metric ρ , except for 0. For k a positive integer such that $k \leq |R(X, \rho)|$, the *Babai k-spectrum* of (X, ρ) is $Spec_k(X) = \{\chi(X, D) \mid D \subset R(X, \rho) \text{ and } |D| = k\}$, and the k^{th} *Babai number* of (X, ρ) is $B_k(X) = \sup Spec_k(X)$. (If X is finite, that "sup" is a "max".)

If H is a finite connected simple graph, the usual distance in H, $dist_H$, is a metric on V(H). Letting $\rho = dist_H$ and allowing V(H)to be replaced by H, we have $R(H, \rho) = \{1, \ldots, diam(H)\}$. We are interested in determining $Spec_k(H)$ and $B_k(H)$ for $1 \le k \le diam(H)$, for various H. In this work we give results when H is a path or a cycle.