# Babai Numbers and Spectra of Paths and Cycles 

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Suppose that $(X, \rho)$ is a metric space and $D \subset(0, \infty)$. The distance graph $G(X, D)$ has vertex set $X$ with $x, y$ adjacent $\Leftrightarrow \rho(x, y) \in D$. The chromatic number of $G(X, D)$ will be denoted $\chi(X, D)$.

Let $R(X, \rho)=\rho(X \times X) \backslash\{0\}$ the range of the metric $\rho$, except for 0 . For $k$ a positive integer such that $k \leq|R(X, \rho)|$, the Babai $k$-spectrum of $(X, \rho)$ is $\operatorname{Spec}_{k}(X)=\{\chi(X, D) \mid D \subset R(X, \rho)$ and $|D|=k\}$, and the $k^{\text {th }}$ Babai number of $(X, \rho)$ is $B_{k}(X)=\sup \operatorname{Spec}_{k}(X)$. (If $X$ is finite, that "sup" is a "max".)

If $H$ is a finite connected simple graph, the usual distance in $H$, dist $_{H}$, is a metric on $V(H)$. Letting $\rho=\operatorname{dist}_{H}$ and allowing $V(H)$ to be replaced by $H$, we have $R(H, \rho)=\{1, \ldots, \operatorname{diam}(H)\}$. We are interested in determining $\operatorname{Spec}_{k}(H)$ and $B_{k}(H)$ for $1 \leq k \leq \operatorname{diam}(H)$, for various $H$. In this work we give results when $H$ is a path or a cycle.

