Fractional Open-Locating-Dominating Sets

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Let G be a simple, connected graph. At any point in time, some vertex may harbor an "intruder," which could represent, e.g., a physical intruder in a facility or an error in a multiprocessor system. A detection system modeled on G is a subset of vertices equipped with specialized sensors that can detect the presence or absence of an intruder within a certain region. A multitude of detection system variants have been explored, such as the Identifying Code, whose sensors detect intruders within distance 1, and the Open-Locating-Dominating Set, whose sensors detect intruders within the open neighborhood. In both of these parameters, a detector either exists or does not exist at any particular vertex. In this paper, we introduce a real-valued extension of the Open-Locating-Dominating Set which allows detectors to take on "fractional" values within $[0, 1]$ at any vertex, which can be interpreted as a sensing duty cycle. Similar fractional extensions have been applied to other 0/1 graphical parameters, such as the Identifying Code and the Dominating Set, but to the best of our knowledge this is the first study of fractional variants of Open-Locating-Dominating Sets. We explore the minimization problem of this parameter on several classes of graphs, including paths, cycles, ladder graphs, and a tight bound on the infinite king grid.

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