New bounds on the anti-Ramsey numbers of star graphs via maximum edge q-coloring

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The *anti-Ramsey number* $ar(G, H)$ with *input graph* G and *pattern graph* H, is the maximum positive integer k such that there exists an edge coloring of G using k colors, in which there are no *rainbow* subgraphs isomorphic to H in G. (H is *rainbow* if all its edges get distinct colors). The concept of *anti-Ramsey number* was introduced by Erdős et al. in 1973. Thereafter, several researchers investigated this concept in the combinatorial setting. Recently, Feng et al. revisited the *anti-Ramsey* problem for the pattern graph $K_{1,t}$ (for $t \geq 3$) purely from an algorithmic point of view. For a graph G and an integer $q \geq 2$, an edge q-coloring of G is an assignment of colors to edges of G, such that the edges incident on a vertex span at most q distinct colors. The *maximum edge q-coloring* problem seeks to maximize the number of colors in an edge q-coloring of the graph G. Note that the *optimum value* of the edge q-coloring problem of G equals $ar(G, K_{1,q+1})$. Here, we study $ar(G, K_{1,t})$, the *anti-Ramsey number* of stars, for each fixed integer $t \geq 3$, both from combinatorial and algorithmic point of view. The first of our main results presents an upper bound for $ar(G, K_{1,q+1})$, in terms of number of vertices and the minimum degree of G. The second one improves this result for the case of triangle-free input graphs. Our third main result presents an upper bound for $ar(G, K_{1,q+1})$ in terms of $|E(G_{\leq(q-1)})|$, which is a frequently used lower bound for $ar(G, K_{1,q+1})$ and maximum edge q-coloring in the literature. All our results have algorithmic consequences.

Key Words: Anti-Ramsey Number, Maximum edge q-coloring problem, Approximation algorithm.