Improved Bounds for Permutation Arrays under Ulam Metric

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For permutations (respectively, strings) π and ρ, let the Ulam distance between π and ρ, denoted by $d_U(\pi, \rho)$, be the minimum number of transpositions of single symbols needed to transform π into ρ. It is known that, for permutations (resp. strings) π and ρ of length n, $d_U(\pi, \rho) = n$ - $LCS(\pi, \rho)$, where LCS denotes the *longest common subsequence*. For a set (array) A, $d_U(A) = \min \{ d_U(\pi, \rho) \mid \pi, \rho \in A, \pi \neq \rho \}$. Let $U(n, d) = \max \{ |\text{A}| \mid d_U(A) \geq d$, where A is a set of permutations (respectively, strings) on n symbols }. Similarly, let $V(kn, k, d) = \max \{ |\text{A}| \mid d_U(A) \geq d$, where A is a set of strings of length kn over the alphabet $\Sigma_k = \{1,2, \ldots, \text{k} \}$ with n of each of the k symbols. Work recently has focused on permutation arrays (codes) under the Ulam metric, due to application in error correction in flash memories. Levenshtein proved in 1992 that $U(n, 2) = (n - 1)!$. We prove that for all positive integers n and $d \leq n$, $U(kn, kd) \geq V(kn, k, kd) \cdot U(n, d)$. For $d = 2$, this yields $U(2n, 4) \geq V(2n, 2, 4) \cdot (n - 1)!$ and, for $d = 3$, $U(3n, 6) \geq V(3n, 3, 6) \cdot (n - 1)!$. We give lower bounds for $V(2n, 2, 4)$ and $V(3n, 3, 6)$, for all n $(1 \leq n \leq 16)$, *e.g.*V(30,2,4) $\geq 3,708$ and $V(30, 3, 6) \geq 14,661$. This yields $U(30, 4) \geq 3,708 \cdot 14! \approx 3.23 \cdot 10^{14}$ and $U(30, 6) \geq 5,320,183,680$. We also give other improved bounds for $U(n, d)$. Using the Erdös and Szekeres Theorem (1935), we show that, for all k, $U(n, n - k) = 2$, for all $n \geq k^3 + 1$.

Keywords: permutation codes, Ulam metric, longest common subsequence