Extremal results of transversal critical graphs

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A transversal set of a graph G is a set of vertices incident to all edges of G. The transversal number of G, denoted by $\tau(G)$, is the minimum cardinality of a transversal set of G. A simple graph G with no isolated vertex is called τ-critical if $\tau(G-e) < \tau(G)$ for every edge $e \in E(G)$. For any τ-critical graph G with $\tau(G) = t$, it has been shown that $|V(G)| \leq 2t$ by Erdős and Gallai and that $|E(G)| \leq \binom{t+1}{2}$ by Erdős, Hajnal and Moon. It was then extended by Gyárfás and Lehel to $|V(G)| + |E(G)| \leq \binom{t+2}{2}$. We show a pure combinatorial result that $r|V(G)| + |E(G)| \leq \binom{t+r+1}{2}$ with all extremal graphs characterized, which is stronger than Erdős-Hajnal-Moon Theorem and Gyárfás-Lehel Theorem. In fact, we prove stronger results via spectral radius.

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