Containment Orders – A Lifelong Journey

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Mathematical explorations steer us through fields and forests of applications where science and computing meet discrete structures and combinatorics. Today's excursion takes us on one of the speaker's favorite journeys in the world of algorithmic graph theory – containment orders, treelike orders and their comparability graphs.

An undirected graph G is a containment graph if each vertex v can be assigned a subset $S(v)$ of a set S such that two vertices are adjacent if one of their sets strictly contains the other. Containment graphs are equivalent to the family of comparability graphs, that is, those admitting a transitive orientation. The classical case of containment graphs of intervals on a line correspond to the comparability graphs having partial order dimension 2, which are equivalent to the family of permutation graphs, and have been generalized to orders of dimension at most $2d$ being representable by containment of axis-aligned boxes in R^d.

After navigating the history of containment graphs and orders, we present recent results on *containment orders of paths in a tree* (CPT for short) which also generalize interval containment. In work with Alcón, Gudiño, Gutierrez, and Limouzy, we have shown that an order P is dually-CPT (P and its dual P^d both admit a CPT-representation) if and only if all the orders sharing the same underlying comparability graph as P admit a CPT-representation. With Limouzy, we have also characterized the partial wheels that are CPT graphs, and show that all partial wheels that admit a transitive orientation are CPT graphs.

Finally, we discuss trivially perfect graphs, known to be equivalent to the comparability graphs of orders whose Hasse diagram is a rooted tree. We survey classical and recent results on this fascinating and highly non-trivial graph family. We conclude with pointers to treelike graphs, those whose Hasse diagram can be any tree.

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