G-Designs for the Tadpole Graphs

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For a subgraph G of the complete graph K_n, a G-decomposition of K_n (or a G-design of order n) is a partition of the edge set of K_n into edge-disjoint copies of G. A tadpole $T(m, n)$ is a unicyclic simple graph consisting of an m-cycle glued to an end vertex of the path with $n - m$ edges. In this article, we show that a $T(m, n)$-design of order N exists whenever $N \equiv 0$ or $1 \pmod{2n}$. Furthermore, if n is a prime power, these conditions are necessary.

Keywords: G-decomposition, G-design, ρ-tripartite labeling, tadpoles, unicyclic graphs