The Noncrossing Bond Poset of a Graph

C. Matthew Farmer*, Anderson University SC; Joshua Hallam, Loyola Marymount University; Clifford Smyth, UNC Greensboro

The partition lattice and noncrossing partition lattice are well studied objects in combinatorics. Given a graph G on vertex set $\{1, 2, \ldots, n\}$, its bond lattice, L_G, is the subposet of the partition lattice formed by restricting to the partitions whose blocks induce connected subgraphs of G. We introduce a natural noncrossing analogue of the bond lattice, the noncrossing bond poset, NC_G, obtained by restricting to the noncrossing partitions of L_G.
Both the noncrossing partition lattice and the bond lattice have many nice combinatorial properties. We show that, for several families of graphs, the noncrossing bond poset also exhibits these properties. We present simple necessary and sufficient conditions on the graph to ensure the noncrossing bond poset is a lattice. Additionally, for several families of graphs, we give combinatorial descriptions of the Möbius function and characteristic polynomial of the noncrossing bond poset. These descriptions are in terms of a noncrossing analogue of non-broken circuit (NBC) sets of the graphs and can be thought of as a noncrossing version of Whitney's NBC theorem for the chromatic polynomial. We also consider the shellability and supersolvability of the noncrossing bond poset, providing sufficient conditions for both.
Keywords: posets, graphs, noncrossing partitions, noncrossing bond poset, Möbius function, characteristic polynomial