

Bipartite completion of colored graphs avoiding chordless cycles of given lengths
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We consider a well-known restriction of the graph sandwich problem: Given a graph G with a proper vertex coloring, determine if there is a completion of G (formed by adding edges while maintaining the proper coloring) that has property P . We study completions that are bipartite graphs without induced cycles of prescribed lengths. It is known that deciding whether there is a chordal bipartite completion is NP-complete when the input graph is colored with an arbitrary number of colors. We consider the case when the input graph is 3-colored and show that the problem of deciding whether there is a bipartite completion that avoids induced cycles $C_6, \dots, C_{2p}$, for fixed $p \geq 3$, is NP-complete. We characterize chordal bipartite completions of 3-colored graphs, and based on this, show that deciding whether a 3-colored graph can be completed to be chordal bipartite is solvable in $O(m + n\alpha(n))$ time. When a chordal bipartite completion exists, a size- n representation of a completion can be constructed within the same time bound. It follows from our results that for every fixed $k \geq 3$, and for every fixed $p \geq 3$, deciding whether a k -colored graph can be completed to be bipartite and $(C_6, \dots, C_{2p})$ -free is NP-complete. Also, the corresponding graph sandwich problems are hard.

Keywords: graph sandwich problem, chordal bipartite completion