Optimum Branching Systems

Jack Edmonds

By branching *B*, rooted at node *r* in digraph *G*, we mean a spanning tree *B* in *G* such that, for every node *t* of *G*, *B* contains a directed path from *r* to *t*. In other words, for every node *t* of *G* except *r*, there is exactly one edge of *B* directed into *t*.

By branching system, for a given digraph *G* and some "root nodes" *r(i)* of *G*, we mean mutually edge-disjoint branchings in *G* rooted at the nodes *r(i)*.

By optimum branching system (OBS), for given *G* and nodes *r(i)* and a cost for each edge of *G*, we mean a cheapest branching system.

Finding an optimum branching system or determining that there is none is a very deep generalization of the min cost network flow problem. The OBS problem is one of the few problems for which, 60 years ago, we found a polynomial-time algorithm. The only understandable way I know to describe the algorithm is as a special case of the optimum matroid intersection problem. In fact it was, 60 years ago, a main motivation for my studying matroid intersections. In one month, I'll be 90 years old and amazingly there is still no computer implementation of the OBS algorithm or an understandable direct algorithm which does not use matroid intersections. To implement would be a significant computer science achievement.

This is dedicated to my mentors from 60 years ago: Ray Fulkerson, Alan Hoffman, Harold Kuhn, Al Tucker, George Dantzig, and Alan Goldman.