The joint asymptotic normality of Spearman's disarray and number of inversions

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Spearman's disarray is a popular statistic on permutations, defined as $\mathrm{SD}(\sigma) = \sum_{i\in[n]} |\sigma_i - i|$, for a permutation σ of length n. The asymptotic distribution of the Spearman's disarray is known to be Normal. Zeilberger derived explicit expressions for several of the first mixed moments for the pair of statistics $(\mathrm{SD}, \mathrm{INV})$, where $\mathrm{INV}(\sigma)$ is the number of inversions in σ. This provided evidence that the asymptotic joint distribution of these statistics is bivariate normal with correlation $\frac{3}{\sqrt{10}}$. By using an article of Petersen and Guay-Paquet, and further ideas of Petersen, we obtained the bivariate generating function of $(\mathrm{SD}, \mathrm{INV})$ as a continued fraction. We will discuss this result, as well as how one can use it to prove rigorously the suggested joint asymptotic normality.

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