Chromatic and achromatic numbers of unitary addition Cayley graphs

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Let R be a finite ring. The unitary addition Cayley graph of R, written $\mathcal{U}(R)$, is the graph with vertex R and two vertices x and y are adjacent if $x + y$ is a unit. We determine a formula for the clique and chromatic number of such graphs when R is a product of local rings. This includes the special case $R = \mathbb{Z}_n$, the integers modulo n, where these parameters had been found only when n is even, or when n is a power of an odd prime. Additionally, we study the achromatic number of $\mathcal{U}(\mathbb{Z}_n)$ in the case that n is the product of two primes. We obtain an exact formula when $n = 3q$ where $q > 3$ is prime, and prove a lower bound that applies when $n = pq$ with $3 \leq p < q$ prime.

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