Super-Extended Split Graphs and the 3-Sphere Regular Cellulation Conjecture

Sergio De Agostino, Sapienza University of Rome, Italy

The 3-sphere regular cellulation conjecture claims that every 2-connected graph is the 1-dimensional skeleton of a regular cellulation of the 3-dimensional sphere, where 2-connectivity is a necessary condition to satisfy such property. The conjecture is obviously true for planar graphs. We introduced a superclass of planar graphs, called the class of extended split graphs, and proved the conjecture for it (S. De Agostino, Extended Split Graphs and the 3-Sphere Regular Cellulation Conjecture, Journal of Combinatorial Mathematics and Combinatorial Computing, 108, 113–123, 2019). This is a superclass of split graphs including also Hamiltonian and complete k-partite graphs. We further extend such class, keeping the validity of the conjecture, by introducing a positive integer parameter k and the notion of orientable homotopic disjointness, which define the class of k-extended split graphs. With this parameter, 1-extended split graphs are a superclass of extended split graphs by means of this notion. Then, k-extended split graphs are defined inductively. A graph is super-extended split if it is k-extended split for some k. Super-extended split graphs are, therefore, the new state of the art for the proof of the conjecture.

Keywords: 3-Sphere, CW-Complex, Regular Cellulation, Split Graph