

On decompositions of 3-uniform hypergraphs into tight cycles

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For positive integers t and v , a t -uniform hypergraph K on v vertices has a set V of size v as its vertex set and a set of t -element subsets of V as its edge set. Thus graphs are 2-uniform hypergraphs. If the edge set of K consists of all t -element subsets of V , then K is complete and is denoted by $K_v^{(t)}$. For graphs G and K , it is common to investigate when the edge set of K can be partitioned or decomposed into isomorphic copies of G . In particular, decompositions into m -cycles are quite popular. Among many others, they have been investigated for K_v , $K_v - I$, where I is a 1-factor, $K_{u,v}$, and for λ -fold K_v . For 3-uniform hypergraphs, tight m -cycles correspond well to the concept of m -cycles for graphs. For $m \geq 4$, a 3-uniform tight m -cycle, denoted TC_m , has vertex set $\{v_1, v_2, \dots, v_m\}$ and edge set $\{v_i, v_{i+1}, v_{i+2} : 1 \leq i \leq m-2\} \cup \{\{v_{m-1}, v_m, v_1\}, \{v_m, v_1, v_2\}\}$. Participants in the Illinois State University Math REU for Pre-service and In-service Teachers have investigated, and in most cases settled, the decompositions of various 3-uniform hypergraph structures into tight 6- and tight 9-cycles. Among their investigations are decompositions: of $K_v^{(3)}$, of ${}^\lambda K_v^{(3)}$, of $K_v^{(3)} - I$, where I is a 1-factor, and of the complete 3-uniform “bipartite” hypergraph $L_{v,v}^{(3)}$. In this talk, we focus on TC_9 -decompositions of $K_v^{(3)}$ and give updates on related results.

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