Abstract Jozefien D'haeseleer

Title: Exploring finite geometry through graph theory and vice versa

In this talk I will discuss three distinct but interconnected areas of research that I have investigated in recent years, all linked by the theme of graph theory and finite geometry.

In the first part, we delve into the theory of Cameron-liebler sets. We will focus on Cameron–Liebler sets in projective and polar spaces [2, 3, 4]. In each of these context, Cameron-Liebler sets can be defined by several equivalent definitions. Notably, each such set corresponds to an intriguing set (or equitable partition into two classes) in the associated Grassmann or dual polar graph. We will see how the underlying association schemes are used to find classification results and new constructions. In this talk I will give an overview of the known results on these Cameron-Liebler sets in different settings.

The second part will be about generalized Johnson and Grassmann graphs, within the field of spectral graph theory. A central question is whether a graph is uniquely determined by its spectrum (the eigenvalues of its adjacency matrix). While many graphs are known to have this property, numerous well-known graphs possess cospectral mates (non-isomorphic graphs with identical spectra). This phenomenon appears prominently in highly symmetrical graphs, including Johnson and Grassmann graphs. Using techniques such as switching, A. Abiad, W. H. Haemers, R. Simoens and I found a construction of cospectral mates for several specific generalized Johnson and Grassmann graphs [1].

In the final part, we investigate the chromatic number of geometrically defined graphs. We describe recent work with F. Pavese, P. Santonastase and V. Taranchuk, where improved lower and upper bounds on the chromatic number of (powers of the) Grassmann graph are obtained by using maximum rank-distance codes [5].

References

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[2] P. J. Cameron, R. A. Liebler. Tactical decompositions and orbits of projective groups. Linear Algebra Appl., 46:91–102, (1982).

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[5] J. D'haeseleer, F. Pavese, P. Santonastaso, V. Taranchuk, Chromatic Number of Grassmann Graphs and MRD codes. arXiv:2602.10777, (2026).