

## Prime labelings on a $3 \times n$ grid graph

Matt A. Ollis, Emerson College

Stephen J. Curran\*, University of Pittsburgh at Johnstown

A graph  $G$  is said to have a *prime labeling* if there exists a bijective function  $\ell : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$  such that  $\ell(u)$  and  $\ell(v)$  are relatively prime whenever  $u$  is adjacent to  $v$ . It is conjectured that the  $m \times n$  grid graph has a prime labeling for all positive integers  $m$  and  $n$ . It is known that for any prime  $p$  and any integer  $n$  such that  $1 \leq n \leq p^2$ , there exists a prime labeling on the  $p \times n$  grid graph  $P_p \times P_n$ . Also, it is known that the ladder  $P_2 \times P_n$  has a prime labeling for all positive integers  $n$ . We assume that Goldbach's Even Conjecture and a strengthened variant of Lemoine's Conjecture are true in order to show that the  $3 \times n$  grid graph  $P_3 \times P_n$  has a prime labeling for every positive integer  $n$ . As a result,  $P_3 \times P_n$  has a prime labeling for every positive integer  $n \leq 10^7$ .

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