Color degree matrices and neighborhood (k, λ)-balanced graphs

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Let G be an n-vertex graph with a k-coloring (which need not be a proper coloring). We define *color degree matrix* $D(G)$ to be the $n \times (k+1)$ matrix in which $D_{i,j}$ is the number of color j vertices in the neighborhood of vertex i for $1 \leq j \leq k$ and $D_{i,(k+1)}$ is the color of vertex i. We say a graph property $\mathcal{P}$ is a *neighborhood balanced-color property*, if whenever G_1 and G_2 are k-colored graphs and $D(G_1) = D(G_2)$, then either both G_1 and G_2 have $\mathcal{P}$ or both do not.

In this talk, we define several neighborhood balanced-color properties, and here we give one example. A k-coloring of a graph G is *neighborhood* (k, λ)*-balanced,* if for any two colors and for every vertex v, in the open neighborhood of v, the number of vertices of one color differs from that of the other color by at most λ. Having such a coloring is an example of a neighborhood balanced-color property. Let S be a set of neighborhood balanced-color properties. For any $T_1, T_2 \subseteq S$, where G_1 has exactly the properties in T_1 and G_2 has exactly the properties in T_2, we use color 2-switches to construct a connected graph that has exactly the properties in $T_1 \cap T_2$. We illustrate this theorem with several examples.