

Color degree matrices and neighborhood (k, λ) -balanced graphs

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Let G be an n -vertex graph with a k -coloring (which need not be a proper coloring). We define *color degree matrix* $D(G)$ to be the $n \times (k + 1)$ matrix in which $D_{i,j}$ is the number of color j vertices in the neighborhood of vertex i for $1 \leq j \leq k$ and $D_{i,(k+1)}$ is the color of vertex i . We say a graph property \mathcal{P} is a *neighborhood balanced-color property*, if whenever G_1 and G_2 are k -colored graphs and $D(G_1) = D(G_2)$, then either both G_1 and G_2 have \mathcal{P} or both do not.

In this talk, we define several neighborhood balanced-color properties, and here we give one example. A k -coloring of a graph G is *neighborhood (k, λ) -balanced*, if for any two colors and for every vertex v , in the open neighborhood of v , the number of vertices of one color differs from that of the other color by at most λ . Having such a coloring is an example of a neighborhood balanced-color property. Let S be a set of neighborhood balanced-color properties. For any $T_1, T_2 \subseteq S$, where G_1 has exactly the properties in T_1 and G_2 has exactly the properties in T_2 , we use color 2-switches to construct a connected graph that has exactly the properties in $T_1 \cap T_2$. We illustrate this theorem with several examples.