Multiplicity of the second-largest eigenvalue of graphs

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The *multiplicity of the second-largest eigenvalue* of the adjacency matrix $A(G)$ of a connected graph G, denoted by $m(\lambda_2, G)$, is the number of times of the second-largest eigenvalue of $A(G)$ appears. In 2019, Jiang, Tidor, Yao, Zhang and Zhao gave an upper bound on $m(\lambda_2, G)$ for graphs G with bounded degrees, and applied it to solve a longstanding problem on equiangular lines. We showed that if G is a 3-connected planar graph or 2-connected outerplanar graph, then $m(\lambda_2, G) \leq \delta(G)$, where $\delta(G)$ is the minimum degree of G. We further prove that if G is a connected planar graph, then $m(\lambda_2, G) \leq \Delta(G)$; if G is a connected outerplanar graph, then $m(\lambda_2, G) \leq \max\{2, \Delta(G) - 1\}$, where $\Delta(G)$ is the maximum degree of G. Moreover, these two upper bounds for connected planar graphs and outerplanar graphs, respectively, are best possible. We will discuss general techniques and specific methods we used in the proofs of these results.

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