Permutation-Invariant Parking Assortments

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We introduce *parking assortments*, a generalization of parking functions with cars of assorted lengths. In this setting, there are $n \in \mathbb{N}$ cars of lengths $\mathbf{y} = (y_1, y_2, \ldots, y_n) \in \mathbb{N}^n$ entering a one-way street with $m = \sum_{i=1}^{n} y_i$ parking spots. The cars have parking preferences $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in [m]^n$, where $[m] \coloneqq \{1, 2, \ldots, m\}$, and enter the street in order. Each car $i \in [n]$, with length y_i and preference x_i, follows a natural extension of the classical parking rule: it begins looking for parking at its preferred spot x_i and parks in the first y_i contiguously available spots thereafter, if there are any. If all cars are able to park under the preference list $\mathbf{x}$, we say $\mathbf{x}$ is a parking assortment for $\mathbf{y}$. Parking assortments also generalize *parking sequences*, introduced by Ehrenborg and Happ, since each car seeks for the first contiguously available spots it fits in past its preference. Given a parking assortment $\mathbf{x}$ for $\mathbf{y}$, we say it is *permutation invariant* if all rearrangements of $\mathbf{x}$ are also parking assortments for $\mathbf{y}$. While all parking functions are permutation invariant, this is not the case for parking assortments in general, motivating the need for a characterization of this property. Although obtaining a full characterization for arbitrary $n \in \mathbb{N}$ and $\mathbf{y} \in \mathbb{N}^n$ remains elusive, we do so for $n = 2, 3$. Given the technicality of these results, we introduce the notion of *minimally invariant* car lengths, for which the only invariant parking assortment is the all ones preference list. We provide a concise, oracle-based characterization of minimally invariant car lengths for any $n \in \mathbb{N}$. Our results around minimally invariant car lengths also hold for parking sequences.

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