On Ramsey Chains in Graphs

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A decomposition $\{G_1, G_2, \ldots, G_k\}$ of a graph G is ascending if G_i is isomorphic to a proper subgraph of G_{i+1} for $i = 1, 2, \ldots, k-1$. The well-known 1987 Ascending Subgraph Decomposition Conjecture states that every graph has an ascending subgraph decomposition. One of the major topics in graph theory involving edge colorings takes place in Ramsey theory where typically for each red-blue edge coloring of a given graph, one of two prescribed monochromatic subgraphs occurs. Gary Chartrand introduced the concept of a Ramsey chain in 2023, which involves the existence of monochromatic pairwise edge-disjoint subgraphs in a red-blue coloring of a given graph that satisfies conditions that were initially specified in the Ascending Subgraph Decomposition Conjecture. Our goal is to determine the maximum length of a Ramsey chain among all possible red-blue colorings of G. Results and open questions are presented in this area of research.

Keywords: Graph decomposition, red-blue edge coloring, Ramsey chain.