Spectra of structured sign patterns

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A sign pattern (matrix) has entries in $\{+, -, 0\}$. We investigate eigenvalue properties of the sign pattern $\mathcal{C} = \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{D} & O \end{bmatrix}$ of order $2n$, where $\mathcal{A}$, $\mathcal{B}$ are prescribed $n \times n$ sign patterns, and $\mathcal{D}$ is a positive diagonal sign pattern. Such sign patterns are associated with dynamical systems of second-order ordinary differential equations $\ddot{\mathbf{x}} = A\dot{\mathbf{x}} + B\mathbf{x}$, where A and B are real matrices of order n. The *refined inertia* of C is the nonnegative 4-tuple $\mathrm{ri}(C) = (n_+, n_-, n_z, 2n_p)$ where n_+, n_- are the number of eigenvalues of C with positive and negative real parts (respectively), n_z is the number of eigenvalues of C equal to zero, and $2n_p$ is the number of nonzero pure imaginary eigenvalues of C. The refined inertia of sign pattern $\mathcal{C}$, denoted $\mathrm{ri}(\mathcal{C})$, is the set of all possible refined inertias of matrices having the sign pattern $\mathcal{C}$. For given sign patterns $\mathcal{A}$ and $\mathcal{B}$ where one of them is a negative diagonal sign pattern, results are determined concerning the potential stability and sign stability of $\mathcal{C}$, as well as the refined inertia of $\mathcal{C}$. The set of possible refined inertias of general sign patterns $\mathcal{C}$ for various forms of $\mathcal{A}$ are determined when $\mathcal{B}$ is a positive diagonal pattern.

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