Minimal Zero Forcing Sets

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The standard color change rule for zero forcing states that if a blue vertex has a unique white vertex in its neighborhood, the white vertex becomes blue. A zero forcing set is a subset of the vertices of a graph that can be initially colored blue so that all vertices in the graph eventually become blue after repeated applications of the color change rule. This combinatorial game is often optimized by finding the minimum possible size of a zero forcing set of a given graph, also known as the zero forcing number of the graph. We examine zero forcing sets that are minimal (with respect to inclusion) rather than minimum. In particular, we investigate when a graph can have polynomially or exponentially many minimal zero forcing sets relative to the number of vertices. In addition, results on the maximum size of a minimal zero forcing set (and when this number equals the zero forcing number) will be presented.

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