Minimal 123-forcing matrices

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A permutation of $\{1, 2, \ldots, n\}$ contains a 123-pattern provided it contains an increasing subsequence of length 3 and is 123-avoiding otherwise. Equivalently, an $n \times n$ permutation matrix contains a 123-pattern provided the 3×3 identity matrix I_3 is a submatrix, and is 123-avoiding otherwise. Let A be an $n \times n$ $(0, 1)$-matrix. Then A is a 123-forcing matrix provided every permutation matrix $P \leq A$ contains a 123-pattern, that is, every $n \times n$ 123-avoiding permutation matrix has a 1 where A has a 0, and so A *blocks* all $n \times n$ permutation matrices that avoid the pattern 123. A 123-avoiding blocking matrix must have at least n 0's. The matrix A with n 0's, all in a row or a column, is a 123-forcing matrix (because it blocks all $n \times n$ permutation matrices), but there are other matrices A with exactly n 0's which are 123-forcing matrices. In the spirit of the well-known Frobenius-König theorem, we characterize the $n \times n$ 123-forcing matrices with exactly n 0's.

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