Bounds on the genus for 2-cell embeddings of prefix-reversal graphs

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In this presentation, we provide bounds for the genus of the pancake graph $\mathbb{P}_n$, burnt pancake graph $\mathbb{BP}_n$, and undirected generalized pancake graph $\mathbb{P}_m(n)$. Our upper bound for $\mathbb{P}_n$ is sharper than the previously-known bound, and the other bounds presented are the first of their kind. Our proofs are constructive and rely on finding an appropriate rotation system (also referred to in the literature as Edmonds' permutation technique) where certain cycles in the graphs we consider become boundaries of regions of a 2-cell embedding. A key ingredient in the proof of our bounds for the genus $\mathbb{P}_n$ and $\mathbb{BP}_n$ is a labeling algorithm of their vertices that allows us to find appropriate rotation systems to bound the number of regions of a 2-cell embedding of said graphs. All of our bounds are asymptotically tight; in particular, the genus of $\mathbb{P}_m(n)$ is $\Theta(m^n nn!)$ for all $m \geq 1$ and $n \geq 2$.

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