On the last new vertex visited by a random walk in a directed graph

Calum Buchanan^{*}, Paul Horn, Puck Rombach, University of Vermont

Consider a simple graph in which a random walk begins at a given vertex. It moves at each step with equal probability to any neighbor of its current vertex, and it ends when it has visited every vertex. It is well known that cycles and complete graphs have the property that, for any given starting vertex, such a random walk is equally likely to end at any other vertex. Ronald Graham asked whether there are other graphs with this property. In 1993, Lászlo Lovász and Peter Winkler showed that cycles and complete graphs are the only undirected graphs with this property. We strengthen this result by showing that cycles and complete graphs (with all edges considered bidirected) are also the only directed graphs with this property.

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