Exploring srg$(45, 22, 10, 11)$ and implications for the Ramsey number $R(5, 5)$

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The diagonal Ramsey number $R(5, 5)$ is the least integer n such that every 2-edge-coloring of K_n contains a monochromatic copy of K_5. The best published bounds are $43 \leq R(5, 5) \leq 46$. We investigate the existence of an $R(5, 5)$-good strongly regular graph with parameters $(v, k, \lambda, \mu) = (45, 22, 10, 11)$ (equivalently, a K_5-free simple graph whose complement is also K_5-free); such a graph would imply $R(5, 5) = 46$. We encode the search as a Boolean satisfiability (SAT) problem and, using symmetry reduction, decompose it into 313 SAT encodings, 141 of which we certify as unsatisfiable. Additionally, we introduce a new encoding for enforcing K_5-freeness that runs empirically faster than the standard approach.

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