Existence of generative strong Γ-zonal labelings for various Γ

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In 2014, Australian physicist Cooroo Egan developed a labeling of plane graphs known as a *zonal labeling*. Zonal labelings are strongly connected to edge 3-colorings of cubic maps. Here we study a generalization of zonal labeling. Given a plane graph G and an abelian group Γ, a Γ-*zonal labeling* is a labeling $\ell : V(G) \to \Gamma \setminus \{0\}$ such that the labels of the vertices on the boundary of each region sum to 0 in Γ. The labeling is considered *strong* if the additive order of each label matches the degree of the vertex. The labeling is considered *generative* if the labels used generate Γ. We consider the existence of groups Γ for which there exists a graph $G_\Gamma \in \mathcal{G}$ that is generative strong Γ-zonal, where $\mathcal{G}$ is some family of plane graphs. We find that if $\mathcal{G}$ is the family of 2-connected multigraphs or 2-connected simple graphs, $\mathcal{G}$ contains a suitable graph G_Γ for all reasonable groups Γ. In the case where $\mathcal{G}$ is the family of 3-connected graphs, there are many interesting open questions. We also explore this problem from the dualized perspective of Γ-cozonal labelings. This is joint work with Richard Low.

Keywords: graph labeling, zonal labeling, abelian groups, dual graphs