Coloring, Cozonal Labelings, and Kempe Chains

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The Four Color Theorem is one of the most famous theorems in graph theory. Kempe chains have played an important historic role in studying both region and vertex four-colorings of plane graphs. More recently, a vertex labeling known as a zonal labeling was introduced which has a strong connection to the Four Color Theorem. We dualize zonal labelings into a region labeling which we call a cozonal labeling. The unique perspective offered by cozonal labelings allows us to clearly identify Kempe chains, as well as prove some known properties of Kempe chains from the cozonal perspective.