

Boolean Expressions as Continuous Time Differentiable Dynamical Systems

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The linear time reduction of a Boolean expression to a ground NAND-form logic program is given, such that the expression is satisfiable iff the program has a fixed point. The program is considered as an operator on $\{T, F\}^n$ where n is the number of clauses. An example is presented. Differentiation of Boolean-valued functions of a real variable is shown as a special case of differentiation on convergence spaces. The category of convergence spaces is a Cartesian closure of the category of topological spaces. This role in formulating differentiation on convergence spaces is briefly described. (It is not necessary to be familiar with these category-theoretic notions for this talk.) The differentiation of a NAND-form logic program clause with respect to the real numbers is shown; the result is a differential-valued function from $\{T, F\}^n$ to $\text{DIFF}(\mathbf{Reals}, \{T, F\})$. Results of computational experiments are shown as trajectories in several examples of these dynamical systems.

Keywords: format, size, information content